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LEGAL AND MATHEMATICAL ASPECTS OF CUMULATIVE VOTING

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Cumulative voting is a development of corporate voting, and was innovated for the purpose of giving minority stockholders the opportunity to be represented on the board of directors.[1] Under the common law each stockholder voted per capita and not according to the number of shares he owned.[2] But this rule no longer applies in modern corporate voting.[3] At present, in the absence of contrary regulations, it is generally held that shares of stock constitute the basis for voting. This rule is obviously much more just than the earlier one, as it permits those who have the largest monetary interest in a corporation, and who stand to lose the most by its improper management, to elect a majority of directors, and thereby to gain a controlling interest.

Cumulative voting gives a stockholder one vote for each share of stock which he owns multiplied by the number of directors to be elected, which votes may be cast for one candidate or dispersed among several. Thus, a mass of votes is placed in the hands of a minority interest, which mass may be concentrated on a minority candidate; thereby tending to permit minority representation commensurate with the minority capital interest. It is practicable only where all the directors are to be elected on one ballot, since it is impossible to vote cumulatively on a single proposition. Cumulative voting is a stockholder's right and does not apply to corporations having neither shares of stock nor stockholders.[4] In South Carolina it is used only in the election of directors, trustees, or managers, and is never used for voting on propositions of corporate policy, finance, etc. This seems to be the general rule elsewhere. The case of *State* ex rel. *Springs v. Ellison*[5] aptly illustrates the mechanics of cumulative voting. In this instance a majority group of stockholders controlled 57 shares, and a minority group controlled 39 shares, out of a total of 100 shares of capital stock. The remaining four shares were apparently not voted. There were seven directors to be elected

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1. 5 FLETCHER, CYCLOPEDIA OF CORPORATIONS §2048 (Perm. ed. 1931).
2. 63 A. L. R. 1106; 7 R. C. L. 339.
3. 63 Id. 1107; 7 Id. 340.
4. Note 1.
5. 106 S. C. 139, 90 S. E. 699 (1916), 13 A. L. R. 130.

and eleven candidates, seven of which were favored by the majority and four by the minority. By concentrating its votes on its four candidates, the minority was able to put all of them in office on the first ballot[6], the majority votes being divided among the seven. It may readily be seen, therefore, that this method of voting may become a powerful weapon in the hands of a minority group of stockholders when the majority is divided or has spread its votes over a comparatively large number of candidates, as was done here.

In the United States the right of cumulative voting is required by constitutional provision in some states.[7] In others it is required by statute.[8] But in some states it is merely made permissive by statute

6. This election was upset on a second ballot which was cast after the minority group had withdrawn from the meeting, and the litigation over the outcome of this second ballot involved issues not pertinent to the topic of this article. An apt statement concerning situations analogous to that portrayed in the *Ellison* case may be of interest at this juncture. In 2 Cook, A TREATISE ON THE LAW OF CORPORATIONS, §609-a, p.p. 1781-1782 (7th ed. 1913) it is said: "There are certain dangers about this mode of voting, and an unwary majority may find that a smart minority has deprived the majority of the control." And in the note, p. 1782, this statement is furthered as follows: "Thus, suppose there are 1,000 shares, and ten directors to be elected, and one person holds 600 shares. Clearly he should be able to elect a majority of the ten directors. Suppose he votes his 600 votes for six of his friends (A, B, C, D, E, and F) and for four of the minority (G, H, I, and J); and suppose at the same time the 400 shares of the minority are cumulated on three other parties (K, L, and M), with ten votes for the four directors mentioned above (G, H, I, and J). The result will then be as follows:

A, B, C, D, E, and F	have	600	votes each
G, H, I, and J	"	610	" "
K, L, and M	"	1,320	" "

"In other words, the minority have secured a majority of the directors. Again, suppose the holder of the 600 shares does not vote for any minority candidate at all, but casts 600 votes for each of his six candidates, A, B, C, D, E, F. Even then he may lose the election. The minority 400 may cumulate their 4,000 votes on six candidates, and give each of the six 666-2/3 votes. Under the cumulative system the majority, in order to be safe, must not only abandon the idea of electing the whole board, but must cumulate their votes on such a proportion of the board as their stock bears to the whole stock, and must not cast complimentary votes for representatives of the minority."

7. E. g., ILL. CONST., Art. XI, §3.

8. E. g., THROCKMORTON'S OHIO CODE ANN., §8623-(50) (1940).

In Young, *The Case for Cumulative Voting*, WISCONSIN LAW REVIEW, Vol. 1950, No. 1, p.p. 49, 54 (Jan. 1950), statistics as of 1948 are cited which show that 20 states have made cumulative voting mandatory either by constitution or by statute, and that 17 others have made it permissive if the articles of incorporation require it. The states in which it is mandatory are: Arizona, Arkansas, California, Idaho, Illinois, Kansas, Kentucky, Michigan, Mississippi, Missouri, Montana, Nebraska, North Dakota, Ohio, Pennsylvania, South Carolina, South Dakota, Washington, West Virginia, and Wyoming. The states in which it is permissive are: Colorado, Delaware, Florida, Indiana, Louisiana, Maine, Maryland, Minnesota, Nevada, New Jersey, New Mexico, New York, North Carolina, Oklahoma, Rhode Island, Tennessee, and Virginia. Also, the National Banking Act has made it mandatory in all national banks since 1933. (12 U. S. C. A. §61.)

whenever the certificate of incorporation or the by-laws so provide[9], and in still others the right does not exist at all. The provision for cumulative voting in the South Carolina Constitution[10] of 1895 reads as follows:

> "The General Assembly shall provide by law for the election of directors, trustees or managers of all corporations so that each stockholder shall be allowed to cast, in person or by proxy, as many votes as the number of shares he owns multiplied by the number of directors, trustees or managers to be elected, the same to be cast for any one candidate or to be distributed among two or more candidates."

A statute which failed to make provision for the election of directors, trustees or managers, pursuant to this section, was, by way of dicta, stated to violate it in *Carolina, etc., Ry. v. McCown.*[11]

And in the case of *Alderman v. Alderman*[12], it was held that a contract, whereby certain stockholders of a corporation transferred their stock to other stockholders, who were to act as trustees and hold the legal title to such stock with full power to vote the same, was not violative of this constitutional provision. Therefore, it would seem to be legal in this state to use stock for voting purposes, the dividends of which are enjoyed by another person.

In the case of *Looker v. Maynard* ex rel. *Dusenbury*[13] the United States Supreme Court declared that a Michigan statute which permitted cumulative voting did not violate the contract clause of the United States Constitution, nor impair the contractual obligations between the State of Michigan and the plaintiff corporation, which obligations were incurred at the date of incorporation. In that case the corporation had been created before the enactment of the challenged statute. But, since the Michigan Constitution had previously reserved to the state the right to alter, amend or repeal future acts of incorpora-

9. E. g., NEW MEX. STAT. ANN., Vol. 4, §54-406 (1941); and FLACK'S ANN. CODE OF MD., Art. XXIII, §24.

10. Art. IX, §11. This section has been executed by statute: §7680, S. C. CODE OF LAWS (1942). An apt statement describing the mechanics of a similar constitutional provision was made by Gordon, J., in Pierce v. Commonwealth, 104 Pa. St. 150 (1883), as follows: "If there are six directors to be elected, the single shareholder has six votes, and, contrary to the old rule, he may cast those six votes for a single one of the candidates, or he may distribute them to two or more of such candidates as he may think proper. He may cast two ballots for each of three of the proposed directors, three for two, or two for one and one each for four others, or finally, he may cast one vote for each of the six candidates."

11. 84 S. C. 318, 327 66 S. E. 418, 426 (1909).

12. 178 S. C. 9, 24; 181 S. E. 897, 903 (1935).

13. 179 U. S. 46 (1900).

tion, the court read this reservation into the contract between Michigan and the plaintiff corporation. The classic case of *Dartmouth College v. Woodward*[14], which previously had unequivocally declared that a state may not by legislation, subsequent to incorporation, impair its contractual obligations with a corporation, was distinguished from the *Looker* case on the ground that the State of New Hampshire had not by its Constitution reserved to itself the right to alter future contracts of incorporation.

Whether and how the rule stated in the *Looker* case would affect corporations organized prior to 1895 has not been passed on in any South Carolina case. The likelihood of any present litigation affecting corporations chartered prior to 1895, as it relates to cumulative voting, is extremely remote. In the first place, the number of corporations organized prior to 1895 and now in existence is bound to be very small. In the second place, because, under the provisions of Article 9, Section 17, of the South Carolina Constitution of 1895, the acceptance by any then existing corporation of any benefits conferred by that Constitution, or by subsequent legislation, "shall be conclusively held an agreement by such corporation to hold its charter and franchises under the provisions of this Article". In other words, any corporation in existence before the adoption of the Constitution of 1895, would, in order to accept the benefit of subsequent laws, have to accept the constitutional requirement of cumulative voting. Manifestly, practically all, if not all, South Carolina corporations now doing business have proceeded under and accepted the provisions of the Constitution of 1895 and of subsequent legislation. In so doing they have likewise subjected themselves to the cumulative voting provision, and, of course, must abide by it.[15]

The foregoing summary of the legal status of cumulative voting in South Carolina naturally leads into a survey of its resulting mathematical complexities and their solution. There are many thousands of corporations in this state, all of which are required by law to hold elections of directors at least once annually, and to elect the "directors,

14. 4 WHEAT. 518, (U. S., 1819).

15. In Commonwealth v. Flannery, 203 Pa. St. 28, 52 A.129 (1902), this precise situation occurred. The Pennsylvania constitution of 1874 required cumulative voting. It also required that no corporation created before the enactment of this constitution could have its charter amended "except upon the condition that such corporation shall thereafter hold its charter subject to the provisions of this constitution." A literary corporation, created in 1870, had its charter amended by a Pennsylvania court in 1892. In 1901, a dispute arose over the validity of certain cumulative votes cast in an election of directors. The court *held* that the corporation, though created prior to the enactment of the constitution of 1874, was bound by its terms, including the provision for cumulative voting, because of the amending of its charter in 1892.

trustees or managers", as the case may be, by means of cumulative voting. Corporate practice for the average South Carolina attorney will probably increase in the future with consequent growing demands upon his skill and resourcefulness in this specialized field. Since problems involving the computation of cumulative votes are almost certain to arise, it would seem to be advisable for him to have at his fingertips a simplified and workable system of solving them by algebra. Since there are few books[16] which deal with this topic, it is the purpose of this article to condense for the reader at least a part of the scant information which is now in print. The chief and most valuable source of information presented here has been the very noteworthy article of Mr. Charles W. Gerstenberg, "The Mathematics of Cumulative Voting".[17]

Nine of the formulae contained therein, as well as two especially prepared for this article by Mr. Charles W. Huff[18], and one prepared by the writer, will be discussed. Though they may at first sight look formidable, the reader is assured that they will be illustrated in the simplest possible terms.

The first five formulae presuppose the existence of the following type of situation: a corporation, with a given number of shares of capital stock outstanding, all of which are to be used for voting purposes in an approaching directors' election, and with only two rival factions among the stockholders, either of which desires to ascertain the number of shares it will need in order to elect a given number of directors out of the total to be elected. Suppose, for example, that a minority faction wished to elect two out of five directors at an approaching directors' election, there being 100 shares of stock outstanding, each share having equal voting power. The following formula may be used:

$$(1) \qquad x = \frac{ac}{b+1} + 1.$$

16. E. g., 1 P-H Corp. Serv. ¶3545 (1947); Dauten, *Business Finance*, PRENTICE HALL, INC.

17. *Gerstenberg, The Mathematics of Cumulative Voting.* JOURNAL OF ACCOUNTANCY (Jan. 1910).
All the algebraic formulae used in this article, except three which will be specifically referred to by footnote, are the original work of Mr. Gerstenberg. The formulae themselves, with the exception of formula 12, have been set forth here exactly as they appear in the original article. But, the illustrative material which demonstrates such formulae is original with the present writer. This is pointed out due to the fact that the JOURNAL OF ACCOUNTANCY requests that no material be used except that which is quoted directly from Mr. Gerstenberg's article. This article by Mr. Gerstenberg is now available at the library of the University of South Carolina Law School.

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The letter *a* represents the total shares of stock, *i. e.,* 100; *b* represents the total number of directors to be elected, *i. e.,* 5; *c* represents the number of directors that the minority faction wishes to elect, *i. e.,* 2; and *x* is the unknown number of shares which the minority must control in order to accomplish this result. Substituting figures for symbols:

$$x = \frac{100 \times 2}{5 + 1} + 1 = \frac{200}{6} + 1 = 33\text{-}1/3 + 1 = 34\text{-}1/3, \text{ or } 34.$$

Since one cannot purchase a third of a share, the fraction may be dropped, and the answer, therefore, is 34.[19] This result may be proved by simple arithmetic. The reader is cautioned, however, always to distinguish between *votes,* on the one hand, and *shares* on the other. It must be remembered that one has as many votes as the number of shares he controls, multiplied by the number of directors to be elected. The total number of shares outstanding is 100, and the total number of votes which these shares can produce is 100 × 5 or 500. The minority group, pursuant to the result reached by formula (1), must purchase enough additional shares to make its total holding 34 shares, which will place in its hands 34 × 5 or 170 votes. The majority faction would not then be in a position to control more than 100 — 34 or 66 shares, which shares could not produce more than 66 × 5 or 330 votes. That the two factions would together control the total voting power of the corporation is evident, since the sum of 170 and 330 is 500. The minority group may so divide its votes as to give ½ × 170 or 85 votes to each of its two candidates.

Now, if the majority attempted to defeat one of the minority candidates, by nominating four of its own, it could only give to each of them ¼ × 330 or 82½ votes. Since a vote may not be divided, it could only give two of its candidates 83 votes apiece, and the other two 82 votes apiece. It is evident, therefore, that the minority would be able to elect its two candidates, since the majority could not cast as many as 85 votes for each of its four candidates. If the minority controlled one share less, *viz.,* 33 shares, they would thereby have only 33 × 5 or 165 votes. One of its candidates could only receive 83 votes, and the other, 82. Therefore, it would be defeated in its purpose to elect two out of five directors. This formula, as well as

19. In Mr. Gerstenberg's article *supra,* the fraction is dropped, which method will be used, where applicable, in this article. However, in the short article in 1 P-H CORP. SERV. ¶3545 (1947), it is suggested that the fraction be counted as a whole number. But this is not necessary, except in the case of formulae (11-12), as will be pointed out later.

the four subsequent formulae, could also be used to solve any similar problem by either a minority or majority faction. If the total number of shares outstanding will not be used for voting purposes, then the symbol *a*, which represented such number of shares in formula (1), could be used to represent the total number of shares which are to be voted; but, in any event, *all* of the needed number of shares indicated by the answer must be purchased out of the particular pool of shares represented by the symbol *a*. This is true of all of the first five formulae. However, other formulae (6-12) will be used later in this article to solve problems arising where all of the outstanding shares are not used for voting purposes, and where the shares indicated by the answer may be purchased out of a pool of shares which is separate and distinct from the pool of shares used in solving said formulae. It may be pointed out, however, that the accuracy of formula (1), as well as of the other formulae used in this article, will be impaired if all of the shares represented by the symbol *a*, or by other applicable symbols in subsequent formulae, are not used for voting purposes in a given election, or, if the opposite faction is divided and distributes its votes among a greater number of candidates than was anticipated. In such a case, the faction using these formulae would purchase a greater number of shares than is actually needed. On the other hand, if a greater number of shares than such symbols indicate are used by an opposing faction, then the faction using these formulae will be defeated in its purpose, because the formulae are designed to indicate the absolute minimum number of shares needed for a given purpose. In many instances if the opposing faction uses one share more than the number of shares used as a basis of computation in these formulae, such faction will defeat at least one candidate of the faction using said formulae. However, with this one exception, and omitting of course, the remote possibility that the total number of directors would be lessened, these formulae presuppose the existence of the worst possible situation, and since any other alteration of such situation would be advantageous, it is obvious that the exact number of shares which the faction using such formulae could need will be amassed in its hands. Consequently, it would be a mathematical impossibility for any of its candidates to be defeated in the election.

The second formula may be used in order to ascertain the number of shares one must control in order to elect *all* of the directors:

$$x = \frac{ab}{b+1} + 1. \tag{2}$$

Using the same figures which were substituted in the first formula for the symbols *a, i. e.* 100; and *b, i. e.* 5, and remembering that x here represents the unknown number of shares required to elect all of the directors, we find that

$$x = \frac{100 \times 5}{5+1} + 1 = \frac{500}{6} + 1 = 83\text{-}1/3 + 1 = 84\text{-}1/3, \text{ or } 84.$$

In other words, it would require 84 out of the 100 shares outstanding in order to elect all five of the directors. The reader may prove this formula in the same manner in which the first was proved.

In order to ascertain the number of shares required in order to elect only *one* out of the five directors, and using the same figures which were used in the foregoing formulae to represent the symbols a and b, respectively, a third formula[20] may be used, *viz*:

$$(3) \qquad x = \frac{a}{b+1} + 1.$$

Substituting:

$$x = \frac{100}{5+1} + 1 = \frac{100}{6} + 1 = 16\text{-}2/3 + 1 = 17\text{-}2/3, \text{ or } 17.$$

Here, 17 would be the least number of shares that could be used in order to elect one out of five directors. One share less than this would be the highest number of shares which would fall just short of electing one director, which number could be ascertained by the use of the following formula:[21]

$$(4) \qquad x = \frac{a}{b+1}.$$

Substituting 100 for a and 5 for b, it may be ascertained that:

$$x = \frac{100}{5+1} = \frac{100}{6} = 16\text{-}2/3, \text{ or } 16.$$

Mr. Charles W. Huff[22] has especially prepared for the use of this article a formula which will enable the reader to ascertain a different unknown element, c. In the first four formulae, it may be noted that x, the number of shares required in order to accomplish a given

20. It should be noted that x here represents the number of shares required in order to elect one director.
21. Here x represents the greatest number of shares which would be utterly worthless for voting purposes.
22. See Note 18.

purpose, was the unknown element. The known elements were a, the total number of shares outstanding; b, the total number of directors to be elected; and c, the number of directors which a faction wished to elect in a given instance. But suppose x is known, and c is unknown. In other words, suppose a stockholder does not wish to purchase additional stock, but merely wishes to ascertain how many directors he can elect with the number of shares he already holds? The following formula will solve his problem:[23]

$$(5) \qquad c = \frac{(x-1)\ (b+1)}{a}.$$

To demonstrate this formula, let us assume that a stockholder owns 51 shares out of a total of 100 shares of stock outstanding, and that he wishes to ascertain how many directors he may safely try to elect out of a total of five directors.

Substituting:

$$c = \frac{(51-1)\ (5+1)}{100} = \frac{50 \times 6}{100} = \frac{300}{100} = 3.$$

In this particular case the result is a whole number. But suppose the stockholder owned only 50 shares. Then, the result would involve a fraction, being 2-94/100 directors. Obviously 94/100 of a director could not be elected. However, the fraction indicates an important fact, *viz.* that the stockholder has almost enough shares to elect 3 directors. Speaking graphically, he lacks only 6/100 of a director. When the fraction is a very large one, as is the case here, it indicates that with the purchase of only a few additional shares, the stockholder could elect one more director. If the fraction, however, is small, this would indicate that a considerable number of shares would have to be purchased in order to elect one more director. It also would show that he could elect the number of directors indicated by the whole number preceding the fraction, by a safe margin, and perhaps, that he could even sell some of his shares and still elect such number of directors. In the present instance, assuming that the stockholder owns only 50 shares, which shares will elect only 2-94/100 directors, in order to ascertain how many additional shares he would

23. The letter c in this formula represents the number of directors which a stockholder may elect when he controls a given number of shares, which number is represented by the letter x. The letters a and b represent the same items which they represented in the first four formulae. It is to be noted that formula (5) was derived from formula (1).

need in order to elect three directors, the stockholder could use formula (1), *supra,* and substitute the applicable figures, as follows:

$$x = \frac{100 \times 3}{5+1} + 1 = \frac{300}{6} + 1 = 50 + 1 = 51.$$

Therefore, since the stockholder's present holding is already 50 shares he would need to purchase only one additional share in order to elect three directors. This result may be readily proved to be correct by a glance back at the result of formula (5), where it was learned that three directors can be elected by the use of 51 shares.

A second type of situation may often arise in corporate voting when it is known that all of the outstanding shares in a corporation will not be used for voting purposes at an election. This situation is more apt to occur in the case of a large corporation where there are large numbers of disinterested and widely scattered stockholders, each owning a relatively small number of shares; and it probably will less often arise in the case of a small corporation, where all of the stock is in the hands of a few individuals each of whom takes an active interest in the affairs of the corporation. Suppose a corporation has 500 shares of stock outstanding, most of which are distributed among many small stockholders who are totally disinterested in the outcome of corporate elections. Let us assume, furthermore, that there is a majority faction, which controls 57 shares of this stock, and a minority faction which controls 39 shares.[24] At an approaching director's election, at which seven directors are to be elected, the minority faction wishes to ascertain how many shares of stock it will need in order to elect four directors. It anticipates that the majority faction will also attempt to elect four directors. Remembering that the majority faction will use only 57 shares, and that it will distribute the votes produced by these 57 shares among only four candidates, how many shares must the minority faction control in order to elect its four candidates, and thereby defeat the ambition of the majority group to control the board of directors? This problem may be solved by the use of the following formula:

24. This situation is analogous to that which existed in the case of *State* ex rel. *Springs v. Ellison,* Note 5, except that in the present instance there are 500, rather than 100, shares of stock outstanding, and that the majority faction will concentrate its 57 shares on four, rather than seven, candidates. It may be remembered that in that case the minority faction, with only 39 shares, elected four directors on the first ballot, but only because the majority faction was divided. The reader is cautioned to distinguish between the number of *candidates* which each side nominates, and the total number of *directors* to be elected. The number of candidates, obviously, will in most instances exceed the total number of directors to be elected.

$$(6) \qquad y = \frac{cr + b + 1}{b - c + 1}.$$

In this formula, b represents the total number of directors to be elected; c represents the number of directors whom the minority faction wishes to elect; r represents the number of shares controlled by the majority faction, all of which are to be used for voting purposes; and y represents the unknown number of shares needed by the minority faction. In this particular example, $b = 7$; $c = 4$; and $r = 57$.

Substituting:

$$y = \frac{4 \times 57 + 7 + 1}{7 - 4 + 1} = \frac{228 + 8}{8 - 4} = \frac{236}{4} = 59.$$

According to this finding, the minority faction would have to control 59 shares in order to elect four out of seven directors. Since the majority controls 57 shares, and the minority 39, the sum of which is 96, there are 500 — 96, or 404 unvoted shares which may be purchased. Therefore, the minority would have to purchase 20 of these shares in order to elect four out of seven directors, since 59 — 39 = 20. It is obvious that none of these 20 shares has to be purchased from among the 57 shares controlled by the majority faction, but rather, may be purchased from an independent pool of shares. If they were purchased from among the 57 majority shares, which fact would probably less often occur, then the minority would simply have more shares than it actually needed in order to elect four out of seven directors. This may be illustrated if it is assumed for a moment that the minority does not have any shares at all, and that the majority has 57. If 57 is substituted for the letter a in formula (1), the answer indicates that the minority would need 29 shares in order to elect four out of seven directors. However, if 57 is substituted for the letter r in formula (6), in the identical situation, the answer indicates that the minority would need 59 shares, as has already been shown. The reason for this apparent discrepancy is that in the case of formula (1), all of the 29 shares must be purchased from among the 57 shares controlled by the majority, thereby reducing its shares accordingly; whereas, in the case of formula (6), the 59 shares may all be purchased from an independent pool. In either instance the minority group would have the minimum number of shares required in order to elect four out of seven directors. The same fact concerning formula (6) is also true of formulae (7-12).

In order to ascertain the number of shares which the minority fac-

tion would need, in the situation involving formula (6) in order to elect *all* of the directors, it could use the following formula:

$$(7) \qquad y = br + 1.$$

Remembering that $b = 7$, and $r = 57$, we may solve for y.[25]

Thus:

$$y = 7 \times 57 + 1 = 399 + 1 = 400.$$

If the minority wishes to ascertain the number of shares needed to elect only *one* director in the same situation, and thus defeat the ambition of the majority to elect *seven* directors, it may use the formula:

$$(8) \qquad y = \frac{r}{b} + 1.$$

Since $b = 7$, and $r = 57$, then[26]:

$$y = \frac{57}{7} + 1 = 8\text{-}1/7 + 1 = 9\text{-}1/7, \text{ or } 9.$$

Since 9 shares would be the minimum amount needed in order to place one minority candidate in office, it naturally follows that 8 shares would fail to elect even one director. This fact could be demonstrated by the use of the following formula[27]:

$$(9) \qquad y = \frac{r}{b}.$$

Substituting 57 for r and 7 for b, we find that[28]:

$$y = \frac{57}{7} = 8\text{-}1/7, \text{ or } 8.$$

If the minority faction desired to ascertain the number of directors which it could elect with a given number of shares, it could use the following formula:[29]

$$(10) \qquad c = \frac{(b+1)\ (y-1)}{y+r}.$$

25. The letter y here represents the number of shares which the minority faction needs in order to elect *all* of the directors.
26. Here y represents the number of shares which the minority faction needs in order to elect only *one* director.
27. The writer derived this formula from formula (8).
28. In this instance, y represents the number of shares which will fall just short of electing one director.
29. This formula, which was prepared for this article by Mr. Huff (Note 18), was derived from formula (6).

Suppose the minority faction owned 59 shares, and wished to ascertain how many directors these shares would elect in the situation in which formula (6) was used.

Since $b = 7$, $r = 57$, and $y = 59$, it may be demonstrated that:

$$c = \frac{(7+1)\ (59-1)}{59+57} = \frac{8 \times 58}{116} = \frac{464}{116} = 4.$$

Thus, it is shown that the minority faction can elect four directors by the use of 59 shares in the election. Since it has already been pointed out in the problem used to demonstrate formula (6), that it would require 59 shares in order to elect four directors, and since in that formula the number of *shares* rather than the number of *directors* was the unknown element, it is evident that formulae (6) and (10) mutually prove each other.

In all of the foregoing formulae, only two factions have been involved. But suppose a third faction enters upon the scene and desires to elect one or more directors? Before, presenting the formula that would meet the needs of such a third faction, reference might here be made to an hypothetical situation in which such formula may be used. The situation described in reference to formula (8) provides a suitable background to demonstrate this new formula. It was there learned that if the minority faction controlled only 9 shares, it could elect one out of seven directors, when the majority faction controlled 57 shares. In other words, the minority would be able to amass 9×7 or 63 votes, all of which it would cast for one candidate. The majority faction would be able to command 57×7 or 399 votes, 57 of which it could cast for each of its seven candidates. Obviously, the lone candidate of the minority would defeat one of the majority's seven candidates. But the majority group would be able so to disperse its votes among six candidates as to cast 67 votes for each of three candidates and 66 votes for the other three. Now if a third faction knew the above facts[30], it would be able to defeat the minority candidate, who would have the least number of votes of all seven candidates, by the use of the following formula:

$$(11) \qquad z = \frac{y}{c} + \frac{1}{b}.$$

30. In Mr. Gerstenberg's article, it is declared that the probability of a third faction being able to discover the facts revealed in a situation analogous to the above "is remote almost to the degree of impossibility".

Remembering that $y = 9$; $c = 1$; and $b = 7$; then:[31]

$$z = \frac{9}{1} + \frac{1}{7} = \frac{63+1}{7} = \frac{64}{7} = 9\text{-}1/7 = 10.$$

Therefore the third faction could, with 10 shares, defeat the minority candidate. The result in this simplified type of situation is so obvious that the use of a formula would hardly be necessary, but as such a situation grows in complexity, its use would greatly expedite the computation of the number of shares which a third faction would need in order to defeat one minority candidate. Suppose, for example that the minority faction had purchased enough shares to make its total holdings 59 shares, as was shown by the use of formulae (6) and (10). The minority could elect four out of seven directors,[32] as it would be able to distribute 59×7 or 413 votes among four candidates giving 104 votes to one candidate and 103 votes to each of the other three. The majority could amass 57×7 or 399 votes, and although it could give only 100 votes to each of three candidates, and 99 votes to the fourth, yet it would be able to cast 133 votes for each of three candidates. Thus the minority would place its four candidates in office, as it could give more votes to each of four candidates than could the majority. However, the majority would be able to give many more votes to each of three candidates than the minority could cast for each of its four candidates.

The reason for this is plain, if it be remembered that the minority desired to control the bare minimum number of shares required in order to elect four directors. With all of the foregoing information in mind, a third faction would be wasting its money if it desired to purchase enough unvoted shares in order to overcome one of the three candidates of the majority, since it could, with many less shares, defeat one of the four minority candidates. By the use of formula (11) the third party would learn that:

$$z = \frac{59}{4} + \frac{1}{7} = \frac{413+4}{28} = \frac{417}{28} = 14\text{-}25/28 = 15.$$

31. In reaching the above result it will be noted that the fraction is counted as a whole number. This formula, and formula (12), constitute the only exceptions to the rule, used consistently throughout this article, that such fractions be dropped. See Note 19.

32. Here, the minority faction has actually, through the purchase of additional shares, become a majority faction. However, to avoid confusion of the two, this faction will be referred to, for the remainder of this article, as the minority faction.

The accuracy of this computation may readily be demonstrated, when we remember that, with 15 shares, the third faction could give 15×7 or 105 votes to its sole candidate, which would exceed by one vote the 104 votes which one minority candidate would receive, and by two votes, the 103 votes which each of the remaining three minority candidates would receive. Now, if the minority faction wished to assure the election of its four candidates, not only as against the voting power of the majority, but also as against the intervention of a third faction, it would have to give to *each* of its four candidates a greater number of votes than the majority could give to *each* of three candidates. It could ascertain the number of shares it would need in order to achieve this result, by the use of the following formula:[38]

$$(12) \qquad y = \frac{cr}{b-c} + 1.$$

Here, as in the foregoing formulae, $b = 7$; $c = 4$; and $r = 57$.

Substituting:

$$y = \frac{4 \times 57}{7-4} + 1 = \frac{228}{3} + 1 = 76 + 1 = 77.$$

This number of shares would produce for the minority 7×77 or 539 votes, which number would enable it to cast 135 votes for each of three candidates, and 134 votes for the fourth. Therefore, it is manifest that the third faction would not prevail over any of the four minority candidates, as it would only be able with 15 shares, to give its sole candidate 105 votes, which would be 30 votes less than the number which three of the minority candidates would receive, and 29 votes less than the number which the fourth minority candidate would receive. In fact, the third faction candidate would be utterly defeated in the election, as he would receive 28 votes less than the majority faction could give to each of three candidates. It would seem therefore, that the use of formula (12) would cause the minority to purchase far more than the minimum number of shares required in order to defeat the lone third-faction candidate. However, if the result of this twelfth formula is scrutinized, it will reveal to the careful

33. This formula originally appeared in Mr. Gerstenberg's article as follows: $\frac{cr}{b-c} + 1$. See Note 17. In formula (12) y represents the number of shares which the minority faction needs in order to give to each of its four candidates more shares than each of three majority candidates could receive. If any fraction is obtained by the use of this formula, such fraction should be counted as a whole number. See Notes 19 and 31.

observer an important fact, *viz.*, that this result would enable the minority to elect its four candidates even if the third faction was determined to elect its candidate by purchasing whatever number of shares would be necessary in order to defeat the candidate which would, in any event, receive the lowest number of votes, whatever that number would be. In other words, the use of this formula would give to each of the four minority candidates more votes than any one majority candidate would receive. A glance back at the result of formula (12) will show that the minority, by purchasing the requisite number of shares in order to make its total holding 77 shares, would be able to give, by a small margin, more votes to each of its four candidates than the majority could give to each of three candidates. This fact would assure the election of the four minority candidates as against the ambition of a third faction to defeat the candidate receiving the lowest number of votes, because the three majority candidates would here receive the lowest number of votes.

It has probably occurred already to the thoughtful reader that some of the foregoing formulae could be used to compute on a percentage basis the number of shares required in order to elect directors in varying situations. To illustrate, the reader is referred to the problem solved by formula (1). In that instance it was shown that where a corporation has 100 shares outstanding, and a minority group wishes to purchase enough shares in order to elect two out of five directors, it would need 34-1/3, or, dropping the fraction, 34 shares. Since the total number of shares being voted is 100, it is evident that the above answer may be converted into terms of percentage. Such percentage, once it is ascertained, may be used in any similar situation, regardless of the number of shares which will be voted, just so long as one desires to elect two out of five directors. In this instance, the answer obtained by the use of formula (1) is 34, but when converted into terms of percentage it should be 33-1/3, or 33.3333. This apparent discrepancy is due to the fact that, first of all, the fraction which was dropped in using the formula must be used when the answer is converted into terms of percentage, and, secondly, the unit one which was added to the 33-1/3 shares in the formula must not be added until after the percentage has been taken of a given number of shares.

For example, suppose a corporation has 3,500 shares of stock outstanding. How many of these shares would a minority faction need in order to elect two out of five directors? By taking 33-1/3, or 33.3333 per cent of 3,500, one finds the answer to be 1,166-2/3, or

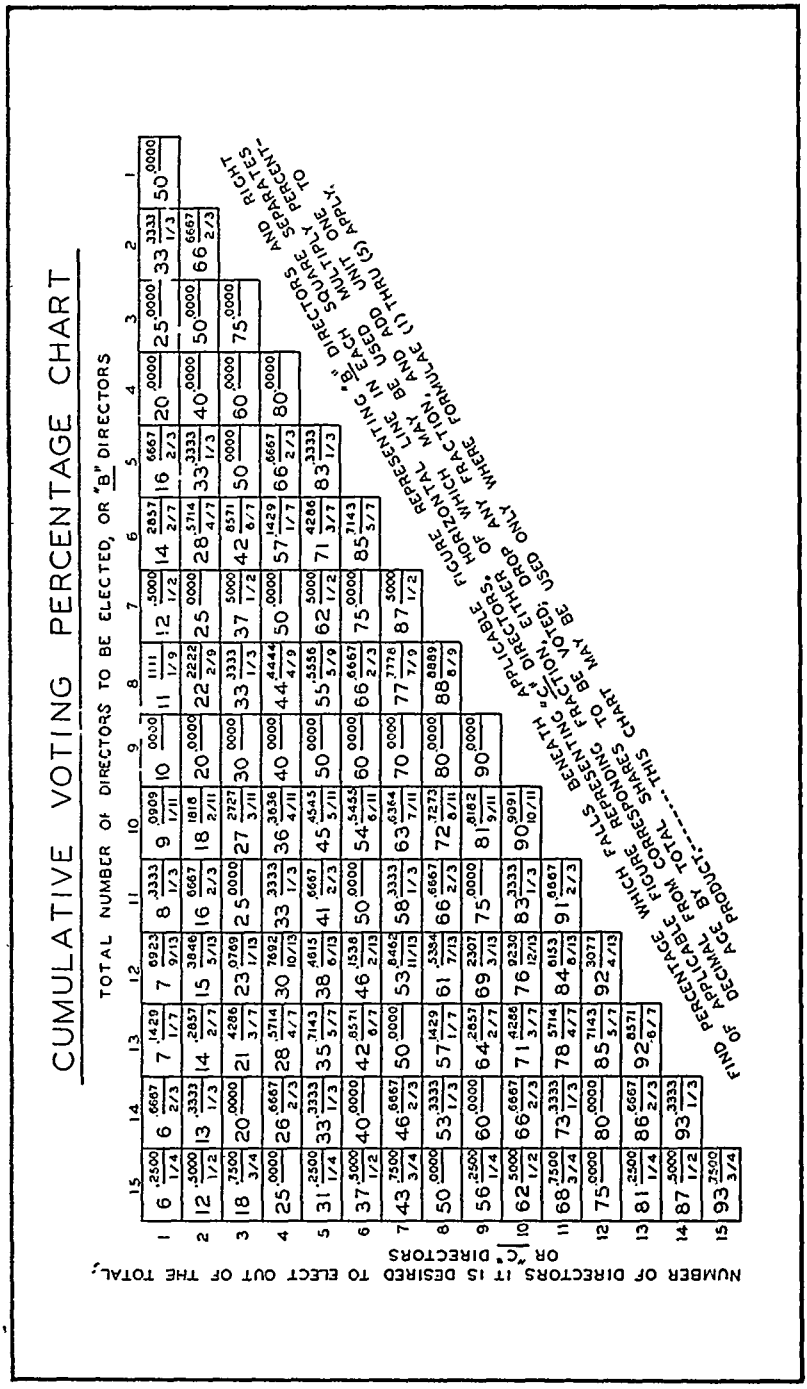
1,166.6667. If the fraction or decimal is dropped and the unit one added to the product, the answer is 167. The same answer is reached by substituting 3,500 for the symbol a in formula (1), and solving for x. Of course, if there were no fraction or decimal in the above answer, the unit one should still be added to the product.

The chart on page 242 shows the percentage of total stock outstanding, or of the total stock to be voted, which will be required in order to elect any given number of directors out of a given total number to be elected; e. g., two out of 13, eight out of 10, etc. It is comprehensive in that it covers every legally possible combination which could arise in South Carolina. The law in this state limits the number of directors which a corporation may have to 15. [34] Consequently, the chart covers all possible elective combinations in this state, which could not be more than 120 in number.

As one faces the chart, it will be noted that on the left-hand side there is a vertical series of figures ranging from one to 15 and increasing in a downward direction; while, at the top, there is a horizontal series of figures, ranging from one to 15, and increasing as one reads from right to left. The vertical row of figures represents the number of directors one wishes to elect out of a given total, expressed by the symbol c in the formulae; whereas, the horizontal row represents the total number of directors to be elected, expressed by the symbol b in the formulae. For example, if one wished to elect three out of seven directors, he should find the square which lies directly to the right of the figure three in the vertical row at the left, and which also falls directly below the figure seven in the horizontal row at the top. This figure is 37 $\frac{.5000}{1/2}$. The decimal ".5000" may be used alternatively with the fraction "½" to which it corresponds. This percentage should then be multiplied by the total number of shares to be voted, any fraction or decimal should be dropped, and the unit one added to the product. The result will be exactly the same as that reached by the use of formula (1).

There are several types of situations in which this chart may be used, but its use is limited to cases where formulae (1-5) apply. *First,* suppose a corporation has 10,000 shares outstanding, all of which are to be used for voting purposes in an approaching director's election, and one who does not own any shares at all desires to purchase enough shares in order to elect one director out of seven. By using either formula (1) or (3), or the chart, it will be found that 1,251

34. §7729, S. C. CODE OF LAWS (1942).


CUMULATIVE VOTING PERCENTAGE CHART
TOTAL NUMBER OF DIRECTORS TO BE ELECTED, OR "B" DIRECTORS
NUMBER OF DIRECTORS IT IS DESIRED TO ELECT OUT OF THE TOTAL OR "C" DIRECTORS
FIND PERCENTAGE FROM FIGURE WHICH REPRESENTS DECIMAL EQUIVALENT OF FRACTION
FIGURE REPRESENTING "B" DIRECTORS AND SEPARATE RIGHT TO BE USED ONLY WHERE FORMULAE (1) THRU (5) APPLY.
HORIZONTAL LINE OF "C" DIRECTORS, EITHER DROP OR ADD ONE
"A" DIRECTORS BENEFITING BY FRACTION, MAY BE USED ANY WHERE
APPLICABLE TO THIS CHART
REPRESENTING FULL SHARES
MULTIPLE VOTE
PERCENTAGE
TOTAL
PRODUCT

is the needed number of shares. However, if one already owns, say, 250 of these shares, this number may be subtracted from 1,251, and only the remaining 1,001 shares need be purchased. *Secondly,* suppose that, in the above situation, there was a majority faction which owned 3,000 shares, all of which would be used in the election, and that the remaining 7,000 outstanding shares were inactive and would not be used. By using either formula (1) or (3), or the chart, and using only the 3,000 majority shares as a basis of computation, it would be learned that, in order to elect one out of seven directors, 376 shares would be needed. All of these shares, however, would have to be purchased from among the 3,000 majority shares. But if one already owned a part of these 3,000 majority shares, for instance, 50 of them, then such number of shares should be subtracted from the 376 needed shares so that only 326 shares would have to be purchased, but the remaining 2,950 majority shares would have to comprise the sole source of such purchase. On the other hand, if the 50 shares already owned formed a part of the 7,000 shares not controlled by the majority, then the total number of shares to be voted would be 3,050. By using 3,050 shares as a basis of computation, the chart, or formula (1) or (3) would reveal that 382 shares would be needed. The 50 shares should be subtracted from 382, leaving only 332 shares to be purchased, but all of these 332 shares would have to be purchased from among the 3,000 majority shares and all of the 382 shares thus amassed would have to be pitted against the remaining 2,668 majority shares. *Finally,* in certain situations two possible methods of computation are left open to the would-be voter. For example, if one did not own any shares at all, and a majority faction owned 3,000 shares which would be voted in an approaching election, leaving 7,000 inactive shares outstanding, formulae (1-5), or the chart, could be used, provided the needed number of shares are purchased from among the 3,000 majority shares; but formulae (6-10) could also be used and the needed number of shares purchased from among the 7,000 inactive shares. Though the needed number of shares indicated by each of the above processes would be different, yet the same purpose would be accomplished if they were purchased from their respective sources. For instance, if one desired to elect two out of seven directors, the use of formula (1), or the chart, would indicate that 751 shares would be needed. But the use of formula (6) would indicate that 1,001 shares would be needed. By purchasing 751 of the 3,000 majority shares, one would then have 751×7, or 5,257 votes, 2,629 of which votes could be cast for one candidate, and 2,628 for the other. The majority, with $3,000 - 751$,

or 2,249 shares, would have 2,249 × 7, or 15,743 votes, 2,623 of which it could give to one candidate, and 2,624 to each of the other five. Therefore, this first method of computation would assure the election of two directors out of seven. On the other hand, if one purchased 1,001 shares, pursuant to the result reached by the use of formula (6), from among the 7,000 inactive shares, he would have 1,001 × 7, or 7,007 votes, 3,504 of which could be cast for one candidate, and 3,503 for the other. The majority, with 3,000 shares, would have 3,000 × 7, or 21,000 votes, only 3,500 of which it could cast for each of six candidates. Hence, this second method of computation would also produce the desired result. Obviously, the use of formula (1), or the chart, would, in the above example, be preferable where all of the 751 needed shares could be purchased from among the 3,000 majority shares. But suppose that only 250 of these 751 shares could be purchased from the majority? Such purchase would leave the majority with only 2,750 shares, which number should be substituted for the symbol r in formula (6), and the answer would be 918. One could then purchase 918 — 250, or 668 of the 7,000 inactive shares, which would be a saving of 1,001 — 918, or 83 shares. But if there were no majority shares available, then it goes without saying that neither the chart nor formulae (1-5) could be used, but formulae (6-10), or in rare instances, formulae (11-12), would have to be used instead.

A final word of caution may be voiced concerning the chart and all of the formulae used in this article. It has already been shown that all of these formulae are designed to ascertain the absolute minimum number of shares needed in order to accomplish a desired end. The same is true of the chart. Consequently, all of such shares must be concentrated on the specific number of candidates indicated by the symbol c in all formulae in which such symbol is used, or by the applicable digits in the vertical row at the left of the chart; and each of such candidates must receive as many of the votes produced by these shares as does any other candidate, or as nearly an equal portion as the division of these votes allows.

Though a percentage chart which may be used alternatively with formula (6) has been devised, yet its use would require such mental gymnastics on the part of the reader as to render its publication unprofitable.